

	a	b	c	d	e	f	g	h
1		4	3				8	
2	2							
3		6			3			
4	5							
5							3	
6								
7		8	1					
8	7							6

Notations:

With (1)...(8) we mean the 1st, ..., 8th row.

With (a)...(h) we mean column a, ..., h.

With (I), (N), ... we mean the pentomino I, N, ...

With a1 we mean the cell in row 1 and column a, etc.

A bold result like **a3=8** means that the cell a3 has to be filled with the digit 8.

A digit group like 732 means these digits in any order. For example, the 3 open numbers of (Y) can be 771 or 762 (filled in anyway).

When we say „because of (a)“, we mean that (a) would contain a digit more than once.

Results:

First we note that the sum of all numbers of a row or a column is 36 (=1+2+3+4+5+6+7+8).

The 3 open numbers of (N) sum up to 15 (= 22 – 2 – 5). This can be done in the following ways:

[1].... 861, 852, 843, 771, 762, 753, 744, 663, 654, 555.

It turns out that only 852 will fit. 6 cannot appear because of (b) and (3), this eliminates 861, 762, 663 and 654. 843 is impossible since because of (b) both digits 8 and 4 would have to be placed in cell a3. For a similar reason, 771, 753, 744 and 555 are impossible.

In case 852, the 8 can only be placed at a3 because of (b), and the 5 can then only be placed at b5. This leaves b4 for the 2: **a3=8, b4=2, b5=5**.

The 4 open numbers of (I) sum up to 15 and must all be pairwise different and also be different from 7 and 6 because of (8). This gives the only possibility:

[2].... (I) contains 87421.

Because of (b) we get **b8=1**, and because of (8) and (g), we get **f8=3** and **g8=5**.

The 3 open numbers of (Y) sum up to 15, so we can check the possibilities from [1]. Because of (1) and (b), 8 cannot appear. 4 cannot appear. Because of (1) and (a), a1 can only be 1 or 6. Case 663 is impossible because only one digit 6 can appear. 555 is clearly not possible. This leaves the two cases 771 and 762 with the following unique placements:

$$[3].... a1=1, d1=7, b2=7 \quad \text{or} \quad a1=6, d1=2, b2=7.$$

In both cases we have $b2=7$. From this follows $b6=3$ because of (b). Now the only place for the 3 in column a is a7: $a7=3$.

Denote region R as (a) & (b) & (c) & d1 & d3. Let's compute the sum of all numbers of R in two ways:

1. (a) + (b) + (c) = $3 \cdot 36 = 108$. So $R = 108 + d1 + d3$.
2. (Y) + (X) + (N) + (U) = $4 \cdot 22 = 88$. So $R = 88 + 3 + 8 + 1 + 7 + 1 + c8 = 108 + c8$.

Comparing these two sums, we get $c8 = d1 + d3$. From [2] we have $c8 = 1, 2, 4$ or 8 . This means $d1+d3 = 1, 2, 4$ or 8 . Clearly, 1 is impossible. 2 is also impossible because then we would get $d1=1$ and $d3=1$, and two 1's in a column are not allowed. 4 would imply $1+3$ or $2+2$, but because of (1) and (3), no cell can contain the 3, and $2+2$ is also impossible as above; there would be two 2's in a column. This leaves only $c8=8$ and $d1+d3 = 8$. The last sum is only possible as $(d1,d3) = (1,7), (6,2)$ or $(7,1)$ because of (1), (3) and (d). Comparing these possibilities with [3] gives the only way to fill the following cells: $a1=1, d1=7, d3=1$.

(a) implies $\{a5,a6\} = \{4,6\}$. The sum $a5+a6+b6$ then is $4+6+3=13$. Therefore the sum $c5+c6 = 22-13 = 9$ because of (U). This can be done as follows, regarding (c), (5) and (6):

$$[4].... (c5,c6) = (2,7), (4,5), (7,2).$$

The two open numbers of (L) sum up to $10 = 22-3-8-1$. Therefore neither of $d5$ and $d6$ can be 5. Because of the other rows, the only place for the 5 in (d) is $d2$: $d2=5$. Then the 3 in (d) can only be at $d4$: $d4=3$. This yields $h2=3$ for the 3 in (h).

Denote region R as (f) & (g) & (h) & e6 & e7. Let's compute the sum of all numbers of R in two ways:

1. (f) + (g) + (h) = $3 \cdot 36 = 108$. So $R = 108 + e6 + e7$.
2. (V) + (Z) + (T) + (W) + (P) = $5 \cdot 22 = 110$. So $R = 110 + f3$.

Comparing these two sums, we get $e6 + e7 = f3 + 2$. Otherwise, because of (W) we get $22 = e6 + e7 + f7 + f8 + g8 = (f3 + 2) + f7 + 3 + 5$. This gives $f3 + f7 = 12$. This can be done as $8+4, 7+5$ or $6+6$. $8+4$ is impossible since 8 cannot appear because of (3) and (7). $6+6$ is clearly not possible because of (f). So we get the two possibilities

$$[5].... (f3,f7) = (5,7) \quad \text{or} \quad (7,5).$$

The 3 open numbers of (F) sum up to $14 = 22 - 5 - 3$. The 3 cannot appear, and because of [5], 5 or 7 must appear. This gives the four possibilities 851, 761, 752 and 554. Checking the placements of the digits, only two possibilities are left:

$$[6].... (e1,e2,f3) = (6,1,7) \quad \text{or} \quad (5,4,5).$$

In either case, the 8 in (2) must appear at $f2$: $f2=8$.

The 8 in (4) can only appear at $e4$ or $h4$. Assume $h4$, then because of (Z) $g2+g3+g4 = 6$, and this cannot be achieved with different digits and without 3. So the 8 in (4) must appear at $e4$: $e4=8$.

The sum of all 64 cells is $8 \cdot 36 = 288$ (the sum of 8 rows) or $12 \cdot 22 + d4+d5+e4+e5 = 264 + d4+d5+e4+e5$ (the sum of the 12 pentominoes and the 4 center cells). This

yields $d_4+d_5+e_4+e_5 = 24$. Since $d_4=3$ and $e_4=8$, we get $d_5+e_5 = 13$. This can be done as $8+5$ or $7+6$. Since 5 cannot appear because of (5), we have $7+6$. Because of (d) we get $d_5=6$ and $e_5=7$. Now the 8 in (d) can only be at d_6 : $d_6=8$. Because of (L) we get $d_7=2$, and again because of (d) $d_8=4$. Now [2] implies $e_8=2$. Then, the 8 in (5) can only be at h_5 : $h_5=8$. And the 1 in (5) can then be only at f_5 : $f_5=1$. This completes (5) with $a_5=4$ and $c_5=2$. From (a) follows $a_6=6$, and then from (U) follows $c_6=7$. The 5 in (c) can only be at c_3 : $c_3=5$. From [5] it follows then $f_3=7$ and $f_7=5$. [6] then yields $e_1=6$ and $e_2=1$. To complete (1) we get $f_1=2$ and $h_1=5$ because of $f_7=5$. To complete (f) we get $f_4=6$ and $f_6=4$ because of $a_6=6$. To complete (e) we get $e_6=5$ and $e_7=4$ because of $f_6=4$. To complete (c) we get $c_2=6$ and $c_4=4$ because of $f_4=6$. To complete (2) we get $g_2=4$. To complete (7) we get $g_7=6$ and $h_7=7$ because of $h_8=6$. To complete (3) we get $g_3=2$ and $h_3=4$ because of $g_2=4$. To complete (4) we get $g_4=7$ and $h_4=1$ because of $h_7=7$. This implies (g) and (h) with $g_6=1$ and $h_6=2$.

	a	b	c	d	e	f	g	h
1	1	4	3	7	6	2	8	5
2	2	7	6	5	1	8	4	3
3	8	6	5	1	3	7	2	4
4	5	2	4	3	8	6	7	1
5	4	5	2	6	7	1	3	8
6	6	3	7	8	5	4	1	2
7	3	8	1	2	4	5	6	7
8	7	1	8	4	2	3	5	6